CSCI 145 Problem 1: The Correlation Floor

October 27, 2026

Submission Instructions

Please upload your work to Gradescope by **11:59pm on Wednesday, September 9**.

- Turn in a single PDF of *handwritten* solutions. Typed (including LaTeX) solutions will not be accepted.
- A complete, good-faith attempt earns submission credit. The grader may select any recent assigned problem for a brief conversation, so keep your work and be prepared to explain each solution and answer a related question. Scheduling details will be announced in class.

Problem 1: The Correlation Floor

A poll asks n people a yes/no question and reports $\bar{X} = n^{-1} \sum_{i=1}^{n} X_i$. Every response has variance $\mathrm{Var}(X_i) = \sigma^2 > 0$. People exposed to the same news or recruited from the same social group may respond alike, so suppose every distinct pair has common correlation ρ, where $0 \leq \rho \leq 1$.

An analyst claims that surveying enough people will make $\mathrm{Var}(\bar{X})$ arbitrarily small, even if the recruitment process is unchanged.

1. What is the exact variance of $\bar{X}$?
2. Does increasing n make this variance arbitrarily small, and how does the answer depend on ρ?

Hint: You may use $\mathrm{Var}(aZ) = a^2 \mathrm{Var}(Z)$ and

$$\mathrm{Var}\left(\sum_i X_i\right) = \sum_i \mathrm{Var}(X_i) + 2 \sum_{i<j} \mathrm{Cov}(X_i, X_j).$$

Takeaway: *With a fixed positive correlation between responses, the uncertainty approaches a positive floor as the poll grows. Reducing that floor requires a sampling process with less dependence between responses.*