CSCI 145 Problem 4: How Many Iterations Does the Power Method Need?

August 27, 2026

Submission Instructions

Please upload your work to Gradescope by **11:59pm on DATE**.

- Turn in a single PDF of *handwritten* solutions. Typed (including LaTeX) solutions will not be accepted.
- Problems are grouped into three-week **cycles**. Once per cycle, you will meet with the grader and present the solution to *one* problem from that cycle, selected at random when you arrive — so bring your work and be ready to explain any of them.

Problem 4: How Many Iterations Does the Power Method Need? 2 points

Let $\mathbf{A} \in \mathbb{R}^{d\times d}$ be symmetric with orthonormal eigenvectors $\mathbf{v}_1, \ldots, \mathbf{v}_d$ and eigenvalues $\lambda_1 > \lambda_2 > 0$ and $\lambda_2 \geq \cdots \geq \lambda_d \geq 0$. The power method starts from the unit vector $\mathbf{v}^{(0)} = \sum_{i=1}^{d} c_i \mathbf{v}_i$ with $c_1 \neq 0$. Let θ_k be the angle between $\mathbf{A}^k \mathbf{v}^{(0)}$ and $\mathbf{v}_1$.

Produce a quantitative guarantee for the number of matrix–vector products needed to make $\tan\theta_k \leq \epsilon$. Your guarantee should account for both the initial alignment $\tan\theta_0$ and the eigengap, and it should be justified from the eigenbasis expansion rather than asserted from the demo's plot. Then evaluate the smallest guaranteed iteration count for $\epsilon = 10^{-6}$ and $\theta_0 = 45°$ in two cases: the lecture matrix with $(\lambda_1, \lambda_2) = (3, 1)$, and a matrix with $\lambda_2/\lambda_1 = 0.99$. Use the comparison to decide when the power method is a practical replacement for a full $O(d^3)$ eigendecomposition, given that each dense matrix–vector product costs $O(d^2)$.

Results you may use. Normalization does not change direction, and lecture established

$$\mathbf{A}^k \mathbf{v}^{(0)} = \sum_{i=1}^{d} \lambda_i^k c_i \mathbf{v}_i.$$

For a vector written in this orthonormal eigenbasis, $\tan\theta_k$ is the norm of its component orthogonal to $\mathbf{v}_1$ divided by the magnitude of its component along $\mathbf{v}_1$.