CSCI 145 Problem 6: A Robust Sensor-Fusion Rule

August 27, 2026

Submission Instructions

Please upload your work to Gradescope by **11:59pm on DATE**.

- Turn in a single PDF of *handwritten* solutions. Typed (including LaTeX) solutions will not be accepted.
- Problems are grouped into three-week **cycles**. Once per cycle, you will meet with the grader and present the solution to *one* problem from that cycle, selected at random when you arrive — so bring your work and be ready to explain any of them.

Problem 6: A Robust Sensor-Fusion Rule 2 points

Several sensors measure the same unknown quantity c. Sensor i reports $y^{(i)}$ and comes with a known scale $s_i > 0$. An engineer is considering two independent-noise models:

$$y^{(i)} = c + \eta_i, \qquad \eta_i \sim \mathcal{N}(0, s_i^2) \quad \text{or} \quad \eta_i \sim \text{Laplace}(0, s_i),$$

where the Laplace density is $p(\eta) = (2s_i)^{-1} \exp(-|\eta|/s_i)$.

Determine the maximum-likelihood fusion rule for c under each model. Your argument should derive the two objectives from their likelihoods and characterize their minimizers, including the possibility that the Laplace minimizer is not unique. In particular, show that the Gaussian rule is a weighted mean and identify its weights, while the Laplace rule is a weighted median: a value for which the total weight strictly below it and the total weight strictly above it are each at most half the total weight.

Apply both rules to

$$(y^{(1)}, y^{(2)}, y^{(3)}) = (0, 2, 100), \qquad (s_1, s_2, s_3) = (1, 1, 10).$$

Then imagine improving only the third sensor, continuously decreasing s_3 from 10. Determine when the Laplace estimate first changes to 100 (and what happens at the boundary), and compare this jump with the way the Gaussian estimate changes. Use the comparison to distinguish resistance to the magnitude of an outlier from sensitivity to its declared noise scale.

Results you may use. The reading's likelihood-to-loss calculation may be quoted. For a weighted absolute-error objective away from the observed values, its derivative is the total weight below the candidate minus the total weight above it.