CSCI 145 Problem 9: The Center Under Four Losses

October 27, 2026

Submission Instructions

Please upload your work to Gradescope by **11:59pm on Wednesday, October 7**.

- Turn in a single PDF of *handwritten* solutions. Typed (including LaTeX) solutions will not be accepted.
- A complete, good-faith attempt earns submission credit. The grader may select any recent assigned problem for a brief conversation, so keep your work and be prepared to explain each solution and answer a related question. Scheduling details will be announced in class.

Problem 9: The Center Under Four Losses

An array of sensors reports $n \geq 1$ readings $y_1, \ldots, y_n \in \mathbb{R}$. We want one number $c \in \mathbb{R}$ to represent them. Different ways of measuring error can favor different notions of a center.

Define the residual vector $\mathbf{r}(c) = (y_1 - c, \ldots, y_n - c)^\top \in \mathbb{R}^n$. Consider these four costs:

$$\|\mathbf{r}(c)\|_0 = \#\{i : y_i \neq c\}, \qquad \|\mathbf{r}(c)\|_1 = \sum_{i=1}^{n} |y_i - c|,$$

$$\|\mathbf{r}(c)\|_2 = \left(\sum_{i=1}^{n} (y_i - c)^2\right)^{1/2}, \qquad \|\mathbf{r}(c)\|_\infty = \max_{1 \leq i \leq n} |y_i - c|.$$

Here # counts elements. The conventional notation $\|\cdot\|_0$ counts nonzero coordinates. It is not a norm.

1. Which constants c minimize $\|\mathbf{r}(c)\|_0$? Prove your answer.
2. Which constants c minimize $\|\mathbf{r}(c)\|_1$? Prove your answer.
3. Which constants c minimize $\|\mathbf{r}(c)\|_2$? Prove your answer.
4. Which constants c minimize $\|\mathbf{r}(c)\|_\infty$? Prove your answer.

Takeaway: *The same readings can have different best centers: a mode, a median, the mean, or the midpoint of the extremes. The answer depends on how we measure error. Choosing a loss decides what we mean by a good prediction.*