CSCI 145 Problem 13: How Shallow Is a Residual Network?

August 27, 2026

Submission Instructions

Please upload your work to Gradescope by **11:59pm on DATE**.

- Turn in a single PDF of *handwritten* solutions. Typed (including LaTeX) solutions will not be accepted.
- Problems are grouped into three-week **cycles**. Once per cycle, you will meet with the grader and present the solution to *one* problem from that cycle, selected at random when you arrive; bring your work and be ready to explain any of them.

Problem 13: How Shallow Is a Residual Network? 2 points

Along one input, the Jacobian of an L-block residual network is

$$\mathbf{J} = (\mathbf{I} + \mathbf{J}_L)(\mathbf{I} + \mathbf{J}_{L-1}) \cdots (\mathbf{I} + \mathbf{J}_1), \qquad \|\mathbf{J}_\ell\|_2 \le \varepsilon < 1.$$

Let $\mathbf{J}_{<m}$ denote the sum of all terms in the product expansion that use fewer than m of the matrices $\mathbf{J}_1, \ldots, \mathbf{J}_L$, in their original order.

A compression memo says that for $L = 50$ and $\varepsilon = 0.1$, "paths of depth at most ten contain at least 99% of the input-output Jacobian." The lecture estimates give the conservative certificate

$$\frac{\|\mathbf{J} - \mathbf{J}_{<m}\|_2}{\|\mathbf{J}\|_2} \le \frac{\displaystyle\sum_{k=m}^{L} \binom{L}{k} \varepsilon^k}{(1-\varepsilon)^L}.$$

Use this certificate to determine the smallest cutoff that guarantees relative error at most 0.01, then issue a verdict on the memo using only the stated norm assumption. Distinguish failure to certify a claim from evidence that the claim is false, support your verdict with an admissible choice of the matrices $\mathbf{J}_\ell$, and explain why "99% of the Jacobian" is potentially misleading when path terms can align or cancel.

The final cutoff should follow from a reproducible calculation, not from a normal or Poisson approximation. You may include a short table of exact binomial sums.

Results you may use. You may use scalar multiples of $\mathbf{I}$ as admissible residual Jacobians, and you may evaluate the exact binomial sums with a calculator or short program.