CSCI 145 Problem 15: Recover a Matrix from a Few Rows and Columns

October 27, 2026

Submission Instructions

Please upload your work to Gradescope by **11:59pm on Wednesday, November 4**.

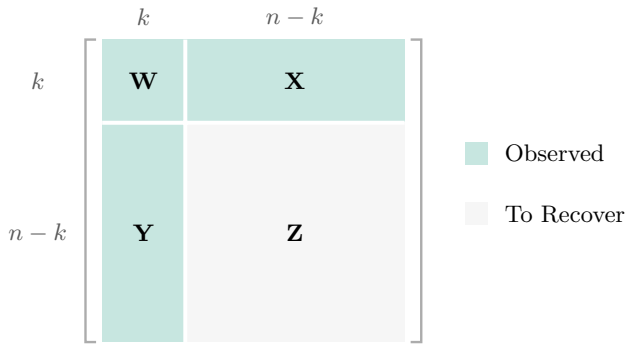
- Turn in a single PDF of *handwritten* solutions. Typed (including LaTeX) solutions will not be accepted.
- A complete, good-faith attempt earns submission credit. The grader may select any recent assigned problem for a brief conversation, so keep your work and be prepared to explain each solution and answer a related question. Scheduling details will be announced in class.

Problem 15: Recover a Matrix from a Few Rows and Columns

A matrix with millions of entries can sometimes be recovered from just a few complete rows and columns. Let $\mathbf{A} \in \mathbb{R}^{n \times n}$ have rank k, where $1 \leq k < n$, and partition it as

$$\mathbf{A} = \begin{bmatrix} \mathbf{W} & \mathbf{X} \\ \mathbf{Y} & \mathbf{Z} \end{bmatrix}.$$

We observe $\mathbf{W} \in \mathbb{R}^{k \times k}$, $\mathbf{X} \in \mathbb{R}^{k \times (n-k)}$, and $\mathbf{Y} \in \mathbb{R}^{(n-k) \times k}$. Assume $\text{rank}(\mathbf{W}) = k$. The remaining block $\mathbf{Z} \in \mathbb{R}^{(n-k) \times (n-k)}$ is unknown.



1. Explain why the columns of $\mathbf{W}$ span $\mathbb{R}^k$ and why the first k columns of $\mathbf{A}$ span its entire column space.
2. Recover $\mathbf{Z}$ exactly in terms of $\mathbf{W}$, $\mathbf{X}$, and $\mathbf{Y}$, and justify your formula.
3. How many distinct entries do we observe? Compare this with the full n^2 entries, and describe the saving when k is fixed and n grows.

Takeaway: *Low rank links the missing entries to the observed ones. When the observed intersection has full rank, a few complete rows and columns determine the entire matrix, even as its dimensions grow.*