# CSCI 145 Problem 16: What Does an Embedding Identify?

August 27, 2026

## Submission Instructions

Please upload your work to Gradescope by **11:59pm on DATE**.

- Turn in a single PDF of *handwritten* solutions. Typed (including LaTeX) solutions will not be accepted.
- Problems are grouped into three-week **cycles**. Once per cycle, you will meet with the grader and present the solution to *one* problem from that cycle, selected at random when you arrive; bring your work and be ready to explain any of them.

## Problem 16: What Does an Embedding Identify? 2 points

A linear autoencoder represents a data matrix $\mathbf{X}$ by codes $\mathbf{Z} = \mathbf{X}\mathbf{W}_0 \in \mathbb{R}^{n\times r}$ and reconstruction $\mathbf{Z}\mathbf{W}_1$. A lab reports that coordinate 7 of its trained code is a meaningful scientific variable. Its reconstruction error is excellent, and a contrastive model trained on the same data gives similar nearest neighbors.

Determine exactly which changes of latent coordinates these observations can and cannot rule out. For an arbitrary invertible $\mathbf{R} \in \mathbb{R}^{r\times r}$, identify a corresponding change to $\mathbf{W}_0, \mathbf{W}_1$ that leaves every reconstruction unchanged. Characterize, with proofs, the matrices $\mathbf{R}$ that preserve for every pair of latent vectors in $\mathbb{R}^r$:

all dot products, all Euclidean distances.

Use your characterization to decide what evidence the lab would need before assigning an interpretation to one coordinate rather than to the geometry of the representation as a whole.

**Optional starred extension** ($\star$)**.** This extension is not required for full credit. For $r \geq 2$, characterize all invertible $\mathbf{R}$ that preserve every cosine similarity. Then determine whether a contrastive objective whose logits are dot products divided by a learned temperature can distinguish every transformation in this larger class.

**Results you may use.** You may use the polarization identity and the fact that $\mathbf{R}$ preserves every Euclidean norm exactly when $\mathbf{R}^\top\mathbf{R} = \mathbf{I}$.